1) Suppose you have a system with an 8 in. by 10 inch monitor that can display 100 pixels per inch. If memory is organized in one-byte words, the starting frame buffer address is 0, and each pixel is assigned one byte of storage, what is the frame buffer address of the pixel with screen coordinates (x, y)?

100 dpi $\Rightarrow$ monitor resolution 800×1000

$\rightarrow x$, $y \downarrow$

0 799

800 1599

1600 . . . .

address $x + y \cdot 800$

2) Describe what the following matrix does:

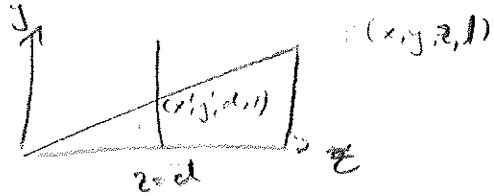
$$\begin{pmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 2 & 0 & 0 \\ -0.707 & 0 & 0.707 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 1 & 0 & 0 \\ -0.707 & 0 & 0.707 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3) Normalize the homogenous point $(2,4,6,2)$

$(1,2,3,1)$

(4) Derive a 4x4 matrix that when applied to the point $\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$ would result in the projection in the following picture:

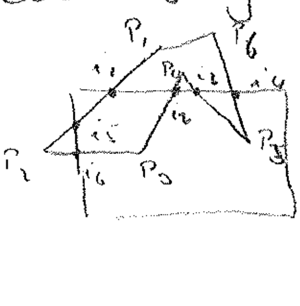


$$\frac{x}{x'} = \frac{z}{d} \Rightarrow x' = \frac{x \cdot d}{z}$$

$$\frac{y}{y'} = \frac{z}{d} \Rightarrow y' = \frac{y \cdot d}{z}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \Rightarrow \begin{pmatrix} x \cdot d/z \\ y \cdot d/z \\ d \\ 1 \end{pmatrix}$$

5) a) Apply the Sutherland-Hodgeman clipping algorithm to the following 2-D polygon. Use the clipping order top, bottom, left, right and give the full list of vertices produced after each stage.



top: i_1 P_2 P_3 i_2 i_3 P_5 i_4
bottom: no changes
left: i_1 i_5 i_6 P_3 i_2 i_3 P_5 i_4
right: no changes

b) Is there anything potentially problematic with the resulting polygon?

Algorithm does not generate two separate polygon but generates two additional segments $i_2 i_3$ and $i_1 i_4$

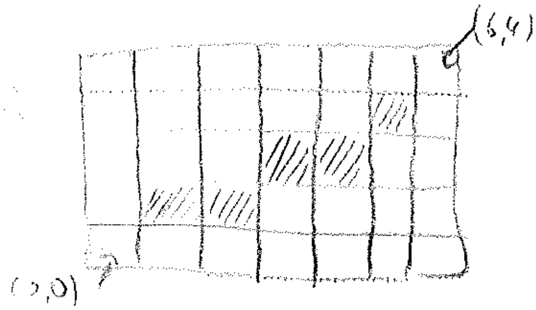
6) Give the inverse of the following matrix:

$$M = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrix describes a translation of $(3, 1, -1)$. Thus, the inverse matrix needs to describe the inverse translation $(-3, -1, 1)$:

$$M^{-1} = \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7) Draw the line specified by its two end points $(1,1)$ and $(5,3)$ using the DDA algorithm.



slope $m = \frac{3-1}{5-1} = \frac{2}{4} = \frac{1}{2} = 0.5$

8) Design a Bresenham-type algorithm (i.e. only using integers) for drawing parabolas of the form $y = \frac{b}{a} \cdot x^2$, $0 \le x \le \frac{a}{b}$

$a>0, b>0$

$0 \le x \le \frac{a}{b} \Rightarrow$ slope $y' \le 1$

$$(x+1)^2 \frac{b}{a} = (x^2 + 2x + 1)\frac{b}{a}$$
$$= x^2 \frac{b}{a} + (2x+1)\frac{b}{a}$$

$$\Rightarrow y_{k+1} = y_k + (2x_k + 1)\frac{b}{a}$$

$$\Leftrightarrow a\, y_{k+1} = a y_k + b(2x_k + 1)$$

$\Rightarrow$ $y_{k+1} = y_k$: $\quad e_{i+1} = e_i + b(2x_k + 1)$

$y_{k+1} = y_k + 1$: $\quad e_{i+1} = e_i + b(2x_k + 1) - a$