



virtual image plane

$$\left. \begin{aligned} e_2 &= u \\ e_1 &= u \times v_d \end{aligned} \right\} \Rightarrow (v_d, e_1, e_2) \text{ view coordinate system}$$

Define distance between eye and image plane, as well as width & height.

Discretize image plane according to the size of your output window

Compute ray through pixel  $(x, y)$ :

$$p = e + d \cdot v_d + \left(x - \frac{x_{\max}}{2}\right) \cdot \frac{w}{\frac{x_{\max}}{2}} + \left(y - \frac{y_{\max}}{2}\right) \cdot \frac{h}{\frac{y_{\max}}{2}}$$

$$v = \frac{p - e}{\|p - e\|}$$

Set up equation for ray:

$$r(t) = e + t \cdot v$$

Intersect with triangle defined by  $v_1, v_2, v_3$ :

$$n_t = \frac{(v_2 - v_1) \times (v_3 - v_1)}{\|(v_2 - v_1) \times (v_3 - v_1)\|}$$

Then, the plane defined by the triangle is:

$$n_t \cdot (x - v_1) = 0$$

Intersection with plane:

$$t = (v_1 - e) \cdot n_t / v \cdot n_t$$

$$\Rightarrow \text{point of intersection: } s = r(t) = e + \left( (v_1 - e) \cdot n_t / (v \cdot n_t) \right) \cdot v$$

check if point of intersection is inside the triangle area of triangle:

$$A = \frac{1}{2} \|(v_2 - v_1) \times (v_3 - v_1)\|$$



other areas:

$$A_1 = A_{v_1, v_2} = \frac{1}{2} \| (v_1 - s) \times (v_2 - s) \|$$

$$A_2 = A_{v_2, v_3} = \frac{1}{2} \| (v_2 - s) \times (v_3 - s) \|$$

$$A_3 = A_{v_1, v_3} = \frac{1}{2} \| (v_1 - s) \times (v_3 - s) \|$$

check if  $A_1 + A_2 + A_3 > A$

Apply Phong illumination model:

Define constant  $I_i, I_c, k_d, k_s, k_a$ , and  $n$

$$L = V = \frac{e - s}{\|e - s\|}$$

$$N = n$$

$$R = (2 \cdot (N \cdot L)) \cdot N - L$$

$$\Rightarrow I = I_i \left( k_d \left( \frac{e - s}{\|e - s\|} \cdot n \right) + k_s \left( \left( 2 \left( n \cdot \frac{e - s}{\|e - s\|} \right) \right) \cdot n - \frac{e - s}{\|e - s\|} \right) \cdot \frac{e - s}{\|e - s\|} + k_a I_c \right)$$

