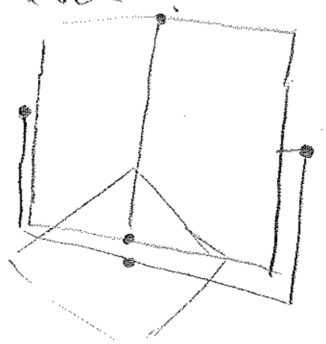
1) a) What problem can occur when using fixed angles?
b) How can it be solved?

a) gimbal lock:



b) use additional rotational axis or quaternions

2) What is the quaternion that represents a rotation of 30 degrees about axis (3, -2, 2)?

rotation as quaternion: $[\cos(\theta/2), \sin(\theta/2)(x, y, z)]$

$s = \cos(15) \approx 0.966$

$v = ((3, 2, 2) / \|(3, -2, 2)\|) \cdot \sin(15)$

$\approx (0.188, -0.126, 0.126)$

3) The interpolation of two quaternions q_a and q_b is given by: $q(t) = (q_a \sin((1-t) \cdot \theta) + q_b (\sin(t \cdot \theta)) / \sin\theta$ where θ is the angle between q_a and q_b.
What is the quaternion that represents the rotation 2/5 of the way between (0.924, (0, 171, 0.342, 0.0)) and (0.866, (0.0, 0.5, 0.0))?

compute θ via the inner product:

$\cos(\theta) = (0.924, (0.171, 0.342, 0.0)) \cdot (0.866, (0.0, 0.5, 0.0))$

$= 0.971$

$\Rightarrow \theta = a\cos(0.971) \approx 13.788$

$2/5 = 0.4$

$\Rightarrow q(0.4) = (((0.924, 0.171, 0.342, 0.0)) \cdot \sin(0.6 \cdot 13.788)$

$+ (0.866, (0.0, 0.5, 0.0)) \cdot \sin(0.4 \cdot 13.788)) / \sin(13.788)$

$= (0.907, (0.103, 0.397, 0.0))$

on (1)

4) For a given camera path p(s), how can we compute the view direction and up-vector?

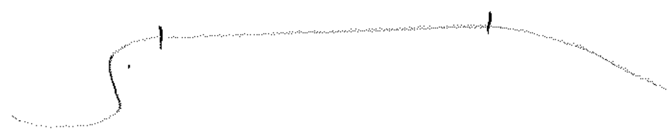
Frenet frame:

view direction: $p'(s)$

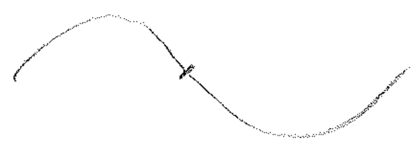
up vector: $p'(s) \times p''(s)$

5) When does the Frenet-frame-based calculation of the upvector fail?

1) when curvature is zero:



2) when second derivative switches direction:



6) What is the difference between global transformations and FFD?

global transformations apply a matrix (or a series of matrices) to an object to deform it. The matrix can be based on the coefficients of the vertex it is applied to. FFD defines deformation by using Bezier-based deformation functional.

7) What is the difference between inverse and forward kinematics?

In forward kinematics, joint parameters are specified explicitly, whereas in inverse kinematics the location of an end effector is specified and the system computes joint parameters.

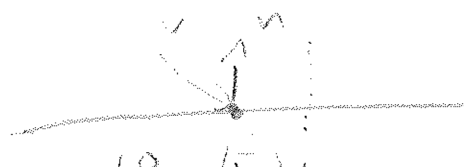
4 :)

8) What methods can help speed up collision detection?

- bounding volumes
  - bounding spheres
  - bounding boxes
  - bounding slabs
  - convex hull
- vertex inside object test

9) Assume a particle is moving towards a horizontal plane with a velocity of $v = \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix}$. Assuming there is no damping, what is the direction of the particle after the collision?

$$n = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



$$v_r = v - 2 \cdot n = \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix} - 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 14 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}$$