

# Computing Center-Lines: An Application of Vector Field Topology

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**Summary.** Curve-skeletons of 3-D objects are medial axes shrunk to a single line. There are several applications for curve-skeletons. For example, the animation of 3-D objects, such as an animal or a human, as well as planning of flight paths for virtual colonoscopy. Other applications are the extraction of center lines within blood vessel where center lines are used to quantitatively measure vessel length, vessel diameter, and angles between vessels. The described method computes curve-skeletons based on a vector field that is orthogonal to the object's boundary surface. A topological analysis of this field then yields the center lines of the curve-skeletons. In contrast to previous methods, the vector field does not need to be computed for every sampled point of the entire volume. Instead, the vector field is determined only on the sample points on the boundary surface of the objects. Since most of the computational time was spent on calculating the force field in previous methods, the proposed approach requires significantly less time while still achieving a better accuracy and robustness compared to methods based on Voronoi tessellations.

## 1.1 Introduction

Curve-skeletons describe the very basic features of an object. They describe a thinned version of the object represented as some type of stick model resulting in the center-lines of the object. Therefore, the use of curve-skeletons can prove useful for applications, such as animation [42] or flight planning for virtual colonoscopy [19]. Similarly, accurate curve-skeleton methods can be used for extracting quantitative measurements from computed tomography (CT) scanned vascular structures. Here, the curve-skeleton describes the center lines of the vessels. These can then be used to measure vessel radius, vessel lengths, and angles between vessels within the volumetric image retrieved by using the CT scanner. This is the application that motivated the development of the algorithm described in this paper. In order to derive these measurements from the volumetric image, an accurate extraction method for curve-skeletons is desirable. For example, thinning-based techniques that work in the voxel space of the volumetric image tend to generate jagged lines which are in no

way suitable for determining angles between vessels. Similarly, inaccuracies can occur when computing the radii of the vessels. Hence, an approach that only uses the volumetric image in order to identify the boundary surface of the contained object is more promising.

The algorithm described in this paper is exactly of this type. It is capable of extracting the boundary surface of an object that is defined by a volumetric image at sub-voxel level. For this, it determines the location of the maximal gradient within the volumetric image similar to Canny's [9] maxima-suppression technique but extended to three dimensions. Since the algorithm only relies on points extracted from the volumetric image but not on the image itself, it can also be applied to object's defined by a point set without any restrictions.

Techniques used for computing the topological graph of a vector field are applied to determine the curve-skeleton. First, for all points vectors are computed that are orthogonal to the object's boundary surface. There are different options for computing these vectors. They can be either derived by determining the normal vector of a plane that is defined by a least-square fit if the point and its neighbors. Or – in case of the object being defined by a volumetric image – the image gradients determined in the previous step can be used. In both cases, the normal vectors can be determined in such a way that they are facing inwards with respect to the object. The entire vector field can then be determined by computing a tetrahedrization of the entire point cloud and then linearly interpolating within the tetrahedra. In order to ensure that only the curve-skeleton in between the object is extracted, all tetrahedra that are located outside the object are removed based on the normal vectors.

A topological analysis of the vector field within the faces of every tetrahedron yields points on the curve-skeleton. By following the topology of the tetrahedrization, points on the curve-skeleton within neighboring tetrahedra can be connected resulting in the entire curve-skeleton.

A detailed description of the algorithm can be found in section 1.3. The next section illustrates related work and compares it to the described approach. Subsequently, the theoretical background with regard to the topological analysis of vector fields is explained. Section 1.4 shows results of the algorithm applied to various data sets, followed by conclusions and future work.

## 1.2 Related Work

Several approaches for extracting curve-skeletons or medial axis can be found in the literature. A very good overview of available techniques can be found in the paper by Cornea et al. [11].

Some methods start with all voxels of a volumetric image and use a thinning technique to shrink down the object to a single line. Directional thinning

approaches use a specific order in which voxels are removed. For example, directions, such as up or down, are used to define this order and conditions are used to identify endpoints [3, 8, 18, 23, 25, 29, 30, 33, 41]. Since these methods are sensitive with respect to the order in which the voxels are removed the resulting curve-skeleton may not be centered. Non-directional methods [6, 38] or fully parallel approaches [14, 26, 28] do not suffer from this disadvantage. Ideally, the topology of the object should be observed. Such an approach was proposed by Lobregt et al. [24] which is the basic technique used in commercial software systems, such as Analyze<sup>TM</sup> developed by the Mayo Clinic. The disadvantage of this approach is that it tends to produce jagged lines which do not allow accurate measurements of angles between parts of the object, such as individual vessels of a vascular structure. Other approaches [39] classify the voxels in different groups, such as edge, inner, curve, or junction and re-classify after removal of a voxel. A similar algorithm is proposed by Palagyi et al. [29]. The disadvantage of thinning algorithms is that they can only be applied to volumetric data sets due to the nature of these algorithms.

To avoid this disadvantage, other approaches deploy the distance transform [17] or distance field in order to obtain a curve-skeleton. For each point inside the object, the smallest distance to the boundary surface is determined. For this, the Euclidian metric or the  $\langle 3,4,5 \rangle$  metric [5] can be used. Also, fast marching methods [35, 40] can be deployed to compute the distance field. Voxels representing the center lines of the object are identified by finding ridges in the distance field. The resulting candidates must then be pruned first. The resulting values are then connected using a path connection or minimum span tree algorithm [37, 43, 46]. Methods used to identify points on the ridges include distance thinning [10, 15, 16, 32], divergence computing [7], gradient searching [4], thresholding the bisector angle [27], geodesic front propagation [31], or shrinking the surface along the gradient of the distance field [34]. The distance field can also be combined with a distance-from-source field to compute a skeleton [47].

Techniques based on Voronoi diagrams [2, 13] define a medial axis using the Voronoi points. Since this approach usually does not result in a single line but rather a surface shaped object, the points need to be clustered and connect in order to obtain a curve-skeleton. Voronoi-based methods can be applied to volumetric images as well as points sets. Due to the fact that clustering of the resulting points is required these approaches lack some accuracy. In addition, they tend to create points outside the object itself if there is an open or missing area within the object's boundary.

### 1.3 Methodology

It is assumed that the reader is familiar with singularities in vector fields and 2-D vector field topology. If necessary, a good overview of these topics can be found in [20, 44, 45].

The algorithm for determining the curve-skeleton consists of several steps. If the object is given as a volumetric image the object's boundary has to be extracted first. Then, a vector field is computed that is orthogonal to the object's boundary surface. Once the vector field is computed, the curve-skeleton can be determined by applying a topological analysis to this vector field. The following subsections explain these steps in detail.

### 1.3.1 Extracting the Boundary of the Object

If the object is given as a set of points, for example measured by a laser scanner, the object's boundary is already defined. If the object is defined by a volumetric image, for example from a CT scan, the boundary of the object has to be determined first. A volumetric image consists of voxels aligned along a regular, three-dimensional grid. Since it is generally not likely that the boundary of the original object is exactly located at these voxels, better precision can be achieved by finding the exact location in between a set of pixels. Since an accurate representation of the object's boundary is crucial to the algorithm, improving the precision is an essential step. The method used within the described system uses similar techniques as described by Canny's non-maxima suppression [9, 21] but extended to three dimensions.

First, the image gradients are computed. Using a fixed threshold, all voxels with a gradient length below this threshold are neglected. Then, the gradients of the remaining voxels are compared to its neighbors to identify local maxima along the gradient. In 3-D, the direct neighborhood of a single voxel generally consists of 26 voxels forming a cube that surrounds the current pixel. In order to find the local maximum along the current gradient, the gradients in the neighborhood in positive and negative direction of the current gradient have to be determined. The current implementation of the described system uses linear interpolation. Based on these three gradient values, i.e. the original image gradient and the two interpolated values, the location of the maximal gradient can be identified, resulting in a more accurate representation of the object's boundary.

Once all points on the boundary are extracted from the volumetric image using this gradient approach with sub-voxel precision, the resulting point cloud can be further processed in order to identify the skeleton.

### 1.3.2 Computing the Vector Field

The described method computes a curve-skeleton by applying a topological analysis to a vector field that is determined based on the geometric configuration of the object of which the curve-skeleton is to be determined. The vector field is computed in such a way that the vectors are orthogonal to the object's boundary surface. The vectors in between the object are then interpolated linearly.

Different approaches are possible for calculating such a vector field. A repulsive force field can be determined that uses the surrounding points on the object's boundary surface as used by Cornea et al. [12]. The repulsive force is defined similarly to the repulsive force of a generalized potential field [1, 22]. The basic idea is to simulate a potential field that is generated by the force field inside the object by charging the object's boundary.

Another way is to define a normal vector by using the neighboring points in addition to the current one and then defining a plane that is approximated by these points. The normal of this plane then defines the vector corresponding to the current point.

If the data is given as a volumetric data set the image gradients can be used to define the vectors on the object's boundary surface. These image gradients are computed already as they are needed for extracting the object's boundary and determining the sub-voxel precision as previously described. Since the points are only moved along the direction of the image gradient when determining the sub-voxel precision, this image gradient is still orthogonal to the object's boundary surface and therefore represents a good approximation for the desired vector field. Note that all three methods result in vectors pointing to the inside of the object.

### 1.3.3 Determining the Curve-Skeleton

In order to determine the curve-skeleton of the object, a tetrahedrization of all points on the object's boundary is computed first. For this, Si's [36] very fast implementation of a Delaunay tetrahedrization algorithm is used. By using the previously computed vectors that point to the inside of the object, outside tetrahedra can be distinguished from tetrahedra that are located inside the object. This way, all outside tetrahedra can be removed, leaving a Delaunay tetrahedrization of the inside of the object only. Since vectors are known for each vertex of every tetrahedron, the complete vector field can be computed using this tetrahedrization by interpolating tri-linearly within each tetrahedron. This vector field is then used to identify points of the curve-skeleton which are connected with each other later on.

Since the vector field is now defined within the entire object, one could use an approach similar to the one used by Cornea et al. [12] at this point and compute the 3-D topological skeleton of the vector field which yields the curve-skeleton of the object. However, since singularities are very rare in a 3-D vector field Cornea et al. had to introduce additional starting points for the separatrices, such as low divergence points and high curvature points, in order to get a good representation of the curve-skeleton. Therefore, a different approach is described in this paper that analyzes the vector field on the faces of the tetrahedra.

In order to be able to perform a topological analysis on the faces of the tetrahedra, the vector field has to be projected onto those faces first. Since tri-linear interpolation is used within the tetrahedra, it is sufficient to just

project the vectors at the vertices onto each face and then interpolate linearly within the face using these newly computed vectors. Based on the resulting vector field, a topological analysis can be performed on each face of every tetrahedron.

Points on the curve-skeleton can then be identified by computing the singularities within the vector field interpolated within each and every face of the tetrahedrization. For example, for a perfectly cylindrical object, the vectors computed at the cylinder's boundary point directly at the center of the cylinder. When looking at the resulting vector field at a cross section of the cylinder, a focus singularity is located at the center of the cylinder within this cross section. The location of this focus singularity resembles a point on the curve-skeleton of the cylinder. Hence, a singularity within a face of a tetrahedron indicates a point of the curve-skeleton. Since the vectors at the object's boundary point inwards, only sinks need to be considered in order to identify the curve-skeleton. Due to the fact that not all objects are cylindrical in shape and due to numerical errors and tolerances, points on the curve-skeleton can be identified by looking for sinks that resemble focus and spiral singularities.

Obviously, only faces that are close to being a cross section of the object should be considered to identify points on the curve-skeleton. In order to determine tetrahedra whose faces resemble a cross-section of the object, the vectors at the vertices can be used. If the vectors at the vertices, which are orthogonal to the object's boundary, are approximately coplanar with the face, then this face describes a cross section of the object. As a test, the scalar product between the normal vector of the face and the vector at all three vertices can be used. If the result is smaller than a user-defined threshold this face is used to determine points on the curve-skeleton. Computing the singularity on one of these faces then results in a point which is part of the curve-skeleton. Since linear interpolation is used within the face, only a single singularity can be present in each face.

Once individual points of the curve-skeleton are computed by identifying the focus and spiral singularities within the faces of the tetrahedra, this set of points needs to be connected in order to retrieve the entire curve-skeleton. Since the tetrahedrization describes the topology of the object, the connectivity information of the tetrahedra can be used. Thus, identified points of the curve-skeleton of neighboring tetrahedra are connected with each other forming the entire curve-skeleton.

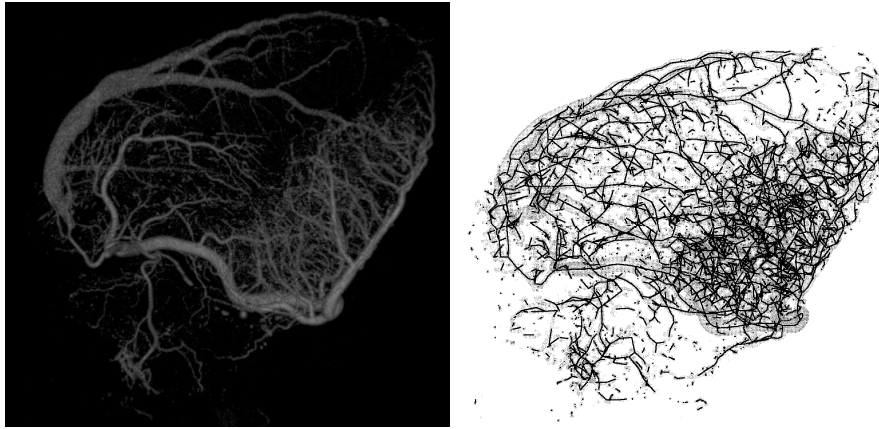
### 1.3.4 The Algorithm

The algorithm for extracting curve-skeletons consists basically of five steps. If the object is given as a volumetric image the object's boundary needs to be extracted from the volumetric image based on a gradient threshold. To increase accuracy, the points are moved along the gradient direction to achieve sub-pixel precision as previously described. Then, vectors orthogonal to the object's boundary surface need to be determined. These can be derived from a

least-square fitted plane of a set of neighboring points and then use its normal vector, or – in the case that the object is defined by a volumetric image – the gradient vectors can be used directly. Subsequently, the point cloud is tetrahedralized so that the resulting tetrahedra can be used to interpolate the vector field using the previously determined vectors at the vertices. Tetrahedra that are located outside the object are not considered when extracting the curve-skeleton. Finally, the topology can be determined on every face resulting in points on the curve-skeleton. By connecting the points found within two neighboring tetrahedra the complete curve-skeleton is generated as a last step.

## 1.4 Results

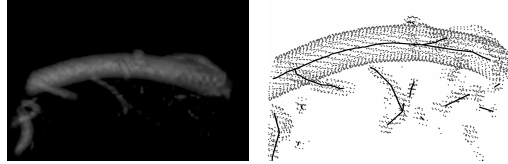
The algorithm was tested on several different data sets. It was mainly designed for extracting center lines on CT scanned images of porcine hearts where the arterials were previously perfused with a contrast enhancing polymer. Figure 1.1(left) shows an example of such a data set.



**Fig. 1.1.** Volume rendering (with shading enabled) of a previously perfused porcine heart which was scanned using a standard hospital CT scanner (left) and curve-skeleton of the porcine heart data set using the described algorithm; to enhance visibility in the paper the thickness of the lines and points within the image is increased (right).

The described algorithm is capable of extracting the curve-skeleton from such a volumetric data set in order to identify the center lines of the arterial vessels. The resulting curve-skeleton is depicted in figure 1.1(right). The figure shows the curve-skeleton extracted by the described algorithm as well as the point set representing the vessel boundaries.

Due to the densely located vessels of the right coronary arterial (RCA) tree, the extracted curve-skeleton seems rather cluttered and it is difficult to distinguish between lines at different depths. However, the extracted curve-skeleton exactly describes the center lines of the arterial vessels found within the data set. When using a sub-section of the porcine heart data set, it can be seen that the curve-skeleton is well defined and located at the center of the arterial vessels, as shown in figure 1.2.



**Fig. 1.2.** Sub-section of the porcine heart data set visualized as a volume rendered image (left) and the extracted curve-skeleton of the same sub-section of the porcine heart data set (right).

The described algorithm for extracting curve-skeletons has some definite advantages over, for example, Voronoi-based approaches. Voronoi-based approaches define a medial axis that is not necessarily a line, but more like a set of points defining a surface. In a post-processing step, these points need to be shrunk down to define a line. Even though the arterial vessels are rather round due to the fact that they are pressurized, a Voronoi-based algorithm only determines a fuzzy line around the actual center line of the vessels. In case the the boundary is not fully defined, meaning that there is a hole in the boundary surface, Voronoi-based approaches tend to move points describing the medial axis outside the object itself, thus far off the center-line. This cannot occur with the described algorithm since center points are only determined for tetrahedra that are located inside the object.

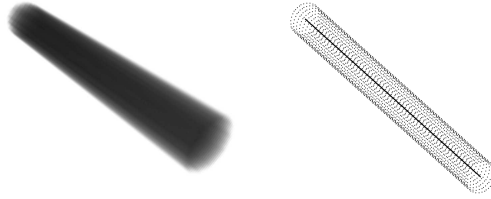
The described algorithm works well with other types of data sets. The first example is a pure cylindrical shaped data set. The cylinder is perfectly round; hence, the algorithm should find a straight line as the curve-skeleton. As can be seen in figure 1.3, the algorithm generates the correct curve-skeleton for this data set.

The second example shows the cow data set (figure 1.4) as a volume rendered image and the point set describing the object's boundary and the curve-skeleton extracted based on these points.

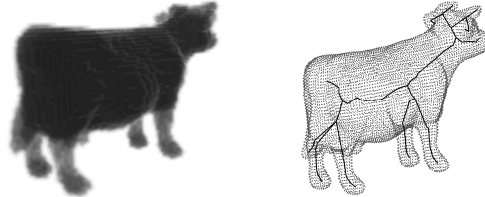
Figure 1.5 depicts the results of the next example, the monster data set. Again, both the volume rendered image and the curve-skeleton including the point describing the object's boundary are shown.

The last example shown in figure 1.6 depicts the mushroom data set. This example indicates that the algorithm can also handle more oddly shaped objects that are not exactly cylindrical shaped.

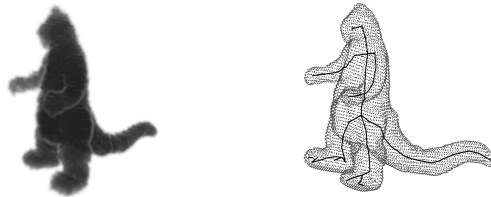




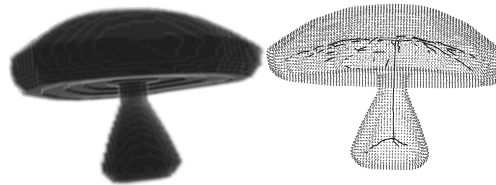
**Fig. 1.3.** Cylinder data set visualized as a volume rendered image (left) and the extracted curve-skeleton of the same data set (right); to enhance visibility in the paper the thickness of the lines and points within the image is increased.



**Fig. 1.4.** Cow data set visualized as a volume rendered image (left) and the extracted curve-skeleton of the same data set (right); to enhance visibility in the paper the thickness of the lines and points within the image is increased.



**Fig. 1.5.** Monster data set visualized as a volume rendered image (left) and the extracted curve-skeleton of the same data set (right); to enhance visibility in the paper the thickness of the lines and points within the image is increased.



**Fig. 1.6.** Mushroom data set visualized as a volume rendered image (left) and the extracted curve-skeleton of the same data set (right); to enhance visibility in the paper the thickness of the lines and points within the image is increased.

## 1.5 Conclusions and Future Work

In this paper, an algorithm for extracting a curve-skeleton from data sets given as point clouds or volumetric images was presented. The described algorithm is based on a topological analysis of a vector field derived from the configuration of the point set describing the object's boundary contained in the data set. Due to the fact that it is no longer necessary to compute the vector field on a multitude of points but instead only for points on the object's boundary the described algorithm is significantly faster while still preserving a high accuracy of the extracted curve-skeleton. It took the algorithm a few seconds to extract the curve-skeleton for the smaller data sets. For the porcine heart data set, the algorithm needed a little more than an hour to determine the curve-skeleton. This is comparably fast, considering that in our tests Cornea's et al. [12] algorithm would have required several months to compute the potential field alone.

It is planned to use the described algorithm for deriving precise quantitative measurements from CT scanned specimens, such as vascular structures. In order to be able to measure vessel lengths, vessel diameters, and angles between vessels an accurate representation of the center lines of the vessels is required. These center lines can be determined by the described algorithm for extracting curve-skeletons applied to the volumetric image generated by a CT scanner.

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